

Scaling Laws for Impact Craters in Water

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INTRODUCTION

Tsunami waves from asteroid impacts into water are of concern from asteroids in the 200 m to 1 km diameter range because this spans the range from asteroids that will likely hit the surface and not airburst, but also be small enough that global climate effects will hopefully be minimal. Current estimates of impact tsunamis depend on either hydrocode simulations or on semi-analytical models. Unfortunately there is significant disagreement between these methods. One of the main reasons for the disparity is that the semi-analytical models such as Holsapple (1993) rely on experimental impacts into deep water. However for asteroids in the 200 – 1000 m range even the deep ocean basins can appear as shallow water impacts where the crater formed in the water reaches all the way to the sea floor. Another reason for the disparity arises from the linear interpolation of data across many orders of magnitude difference between Froude number (ratio of kinetic to gravitational energy) used in the laboratory experiments and what would be seen in an asteroid impact. The Gault & Sonett (1982) experiment shot millimeter sized glass spheres into water at 1 to 6 km/s and the Olevson (1969) experiment dropped millimeter sized water drops at a few meters per second. The goal of this work was to fill in the gaps and conduct experiments and simulations at the correct Froude numbers of interest, and in both deep and shallow water, to help resolve the disparity and extend the semi-analytical models.

Impacts,	Π_g	10^{-8}	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}	$\Pi_g = \frac{1}{2}gD/v^2$	
10m/s,	\emptyset	$0.2 \mu\text{m}$	$2 \mu\text{m}$	$20 \mu\text{m}$	0.2 mm	2 mm	2 cm		Reasonable experiments
100m/s,	\emptyset	$20 \mu\text{m}$	0.2 mm	2 mm	2 cm	20 cm	2 m		
1km/s,	\emptyset	2 mm	2 cm	20 cm	2 m	20 m	200 m		Asteroid Impacts
12 km/s,	\emptyset	0.3 m	3 m	30 m	300 m	3 km	30 km		
20 km/s,	\emptyset	0.8 m	8 m	80 m	800 m	8 km	80 km		
35 km/s,	\emptyset	2.5 m	25 m	250 m	2.5 km	25 km	250 km		
Non-Hazardous		Tsunami Hazard		Global Extinction					
Explosives,	Π_g	10^{-8}	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}	$\Pi_g = \frac{1}{4}gD/E/m$	
5MJ/kg,	\emptyset	2 cm	20 cm	2 m	20 m	200 m	2 km		

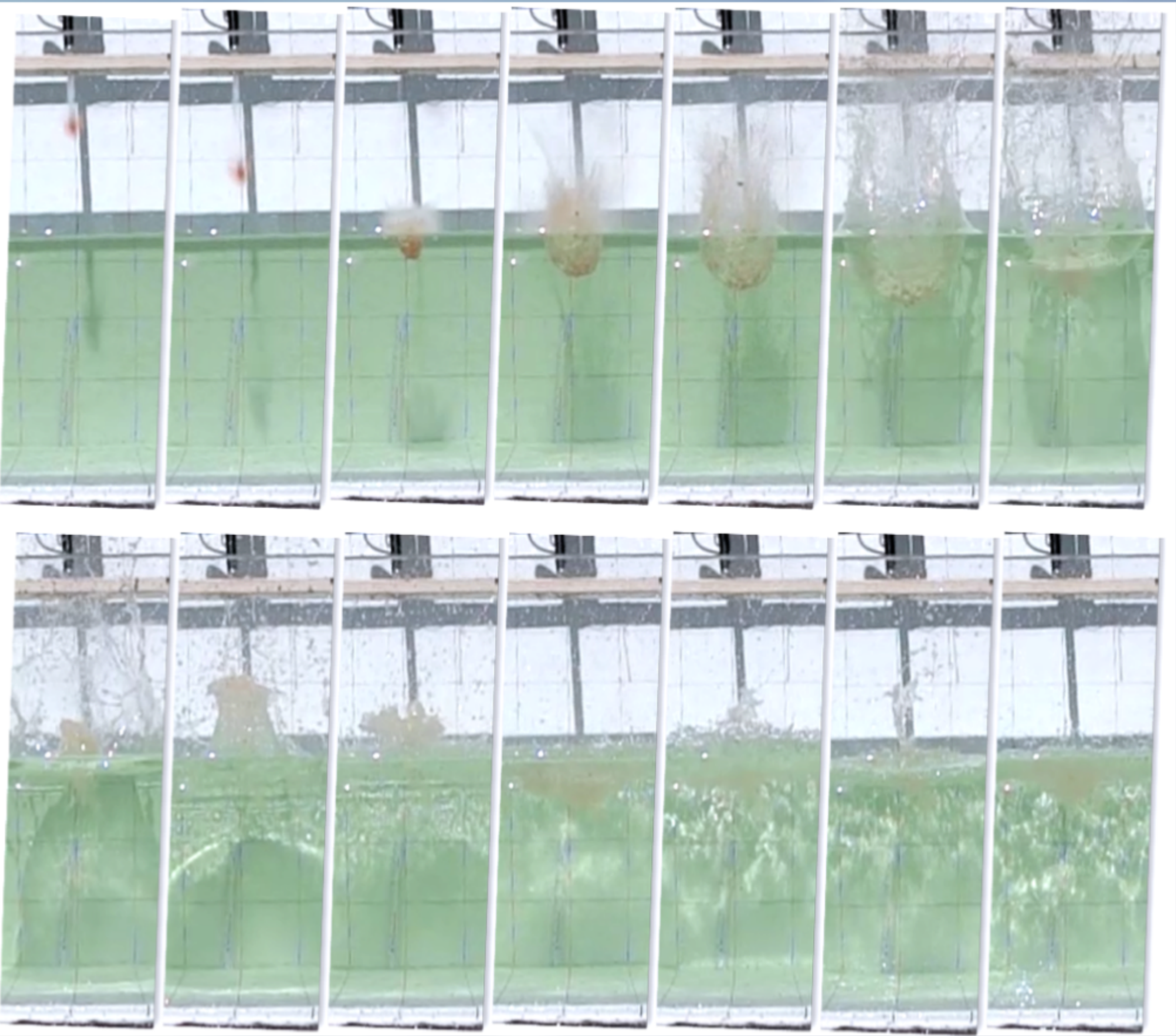
METHOD

Paintballs (1.75 cm diameter shot at 90 m s^{-1}) were determined to be a good scaled model for asteroids of concern for tsunami impacts having a Π_g value of about 5×10^{-6} . A water tank was built of wooden panels 3 foot in length with a 3/8" clear acrylic panel on one side and reinforced by 2x4" corner posts. It was sealed with marine caulk and painted white, with a 6" grid marked on the inside walls. A frame was made of 1x1" wooden studs to support the paintball gun vertically over the center of the tank. Paintballs were individually weighed and measured and inserted into the breach for each shot. After it was determined that the strength of the paintball shells was affecting the results, a razor blade was installed to split the paintballs in half. This was done by mounting a 2x4" board horizontally just below the muzzle and drilling a 1" diameter hole in the center. A small groove was cut, into which the razor blade was inserted. A high speed camera capable of shooting 336 x 96 pixel images at 1200 frames per second was placed on the centerline of the tank and the height and distance recorded for each shot. In most tests the height of the camera was set close to the water level to minimize total internal reflection from the water surface.



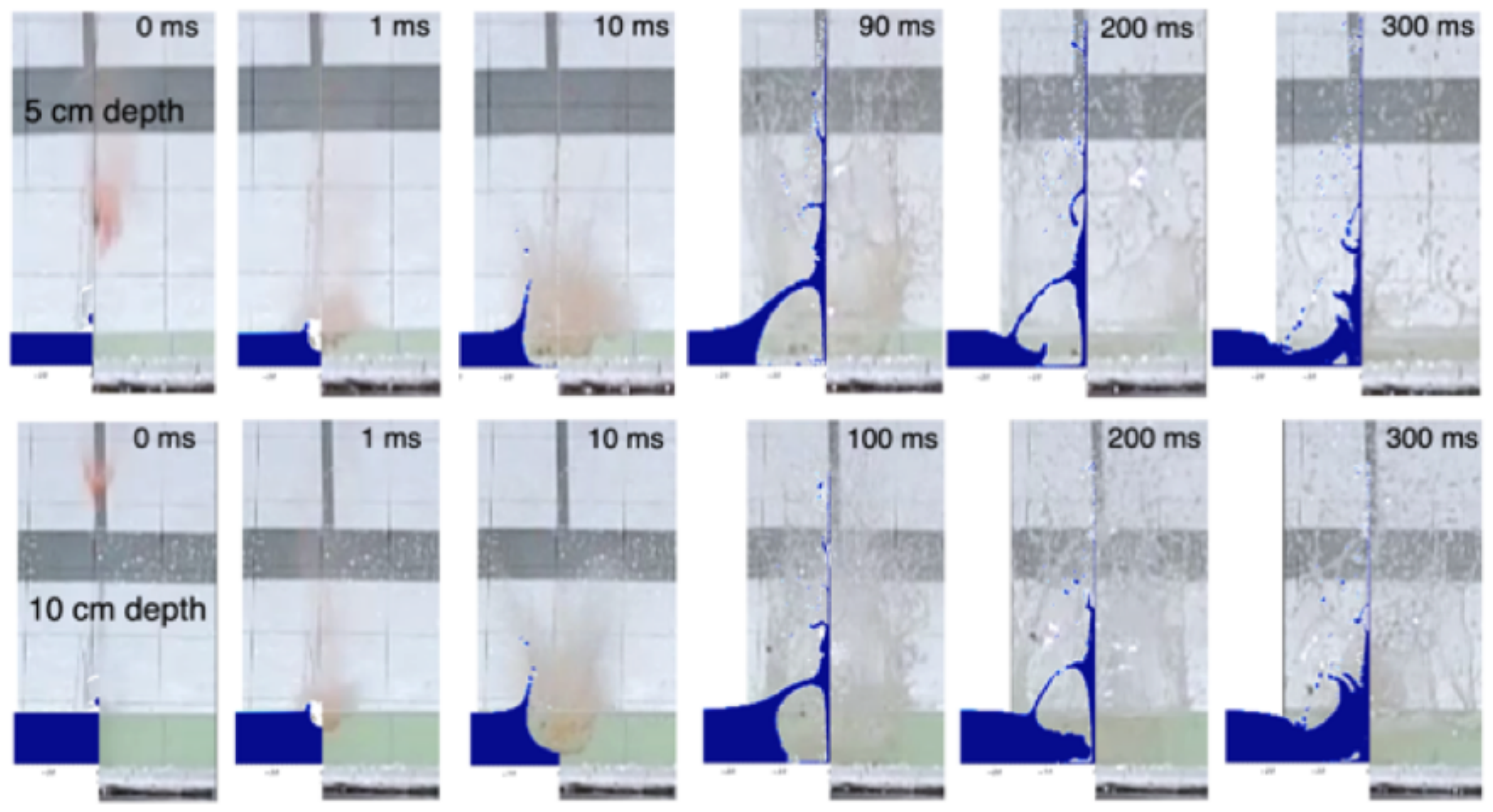
EXPERIMENTAL RESULTS

After being split by the razor, the paintball suffers a little dispersion, but still impacts the water with a much smaller diameter than the size of the crater it will create. Upon impact a hemispherical cavity starts to form while a curtain of splashed water is ejected upwards. After the maximum crater sized is reached, the cavity rebounds into a central jet, which in turn collapses back into another cavity. Oscillation between the cavity and jet pumps out several tsunami waves. For shallow water impacts, initially a roughly hemispherical cavity forms until the tank floor is reached. The maximum crater diameter is only slightly smaller than for impacts into deep water, but the depth is limited by the depth of water.



SIMULATIONS

To increase confidence in the simulations of asteroid impacts, simulations were run of the experiments including our paintballs, the hyper-velocity experiments of Gault & Sonett (1982) and the droplet experiments of Olevson (1969). Simulations below show excellent agreement in crater size vs time. Additional experiments were found in the literature for droplets in a vacuum tower (Engel 1962,1966), droplets into shallow water (Berberović 2009, Hinsberg 2010), small explosives in different depths of water (Kriebel 1969). Simulations then expanded beyond the experiments to investigate the effects of background air pressure, water depth, and asteroids hitting an ideal perfectly hard ocean floor.



SCALING LAWS

Holsapple (1993) provides scaling laws for impacts into a variety of targets which for water is $\frac{m_c}{m_i} = K_1 \left(\frac{1}{2}gD/v^2 \right)^{-0.647} \left(\frac{\rho_c}{\rho_i} \right)^{0.224}$

where m_c is the mass of the material excavated by the crater (in this case water), m_i is the mass of the impactor, D and v are the impactor diameter and velocity, and ρ_c and ρ_i are the crater and impactor densities. For water the craters are hemispherical so the crater mass is $m_c = \frac{1}{2}\pi R^3$.

The experiments and simulations show a strong dependence on background air pressure at low values of $\Pi_g = \frac{gD/v^2}{v_i^2}$, which decreases as Π_g increases. They also show that the crater size starts to decrease as the depth of water becomes shallower, eventually being dominated by the depth. Potential flow theory and considering the work done in expanding the crater against the pressure at a given depth of water yields $\pi \rho_c R^3 \dot{R}^2 + \frac{2}{3}\pi P_0 R^3 + \frac{1}{2}\pi \rho_c g R^4 = E$

where R is the crater radius, \dot{R} is the crater expansion velocity, P_0 is the surface pressure, g is the gravitational acceleration, and E is the energy driving the cavity expansion which can be set equal to the kinetic energy of the impactor, or at least some fraction of it. The maximum crater size occurs when $\dot{R}=0$. Fortunately the non-linear equation in R can be closely approximated by

$$R^3 = \frac{1}{\frac{2}{3}\pi P_0/E + \left(\frac{1}{2}\pi \rho_c g/E \right)^{3/4}}$$

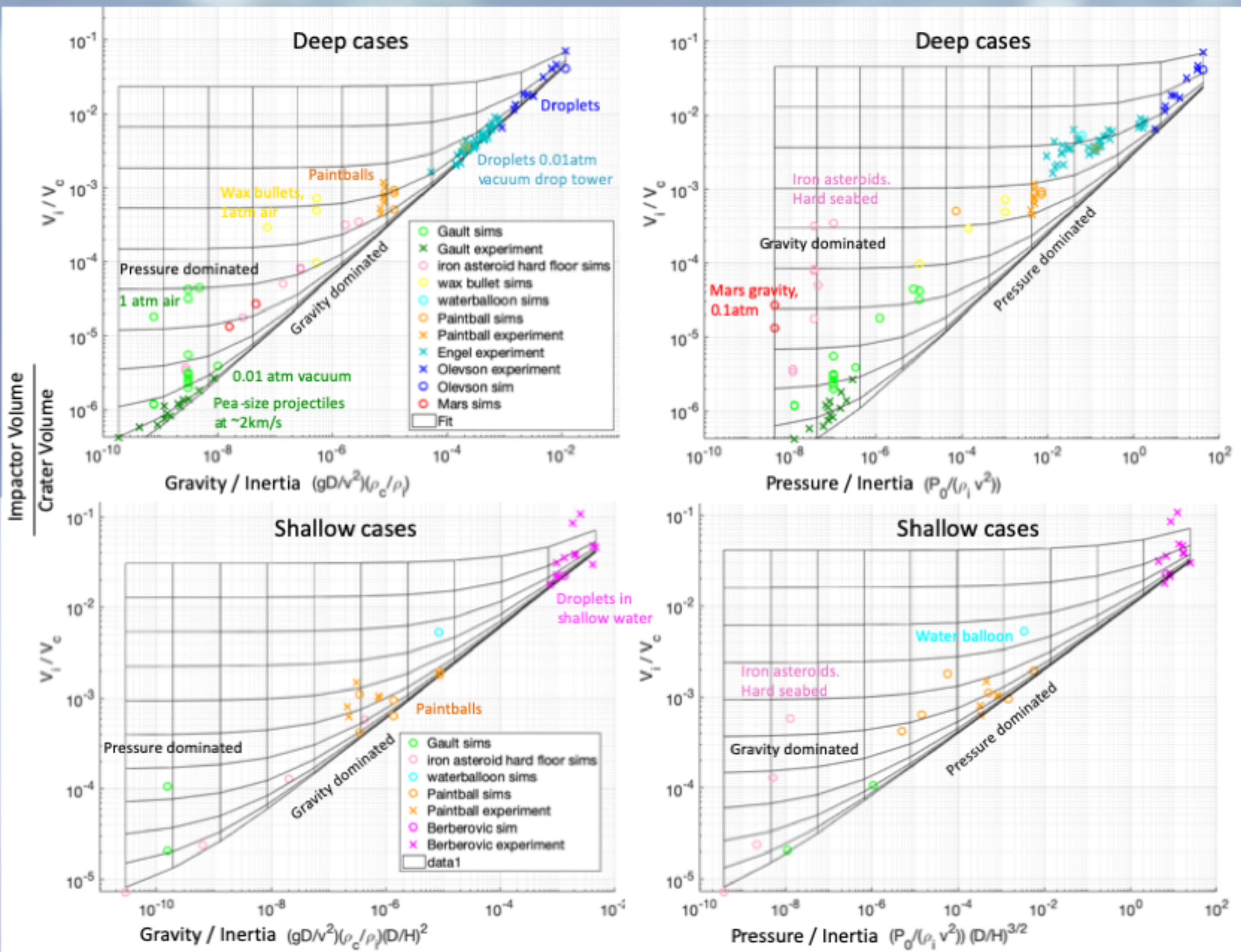
To be useful as a scaling law, this equation can be non-dimensionalized by considering the ratio of crater volume to impactor volume. In shallow water the crater radius was seen to be approximately that of the deep water impacts, but the depth will be the water depth, suggesting a scaling law of the form

$$\frac{V_i}{V_c} = D^3 \left(\frac{1}{6H} \left(\frac{2}{3}\pi P_0/E \right)^{2/3} + \frac{1}{6H} \left(\frac{1}{2}\pi \rho_c g/E \right)^{1/2} + \frac{\pi P_0}{6E} + \frac{1}{4} \left(\frac{1}{2}\pi \rho_c g/E \right)^{3/4} \right)$$

where V_i and V_c are the impactor and crater volumes, and H is the water depth.

In general, only a fraction of the impactor kinetic energy will go into driving the cavity so the constants will be different, and since that fraction may not be constant across the entire range the exponents may be different too, but the non-dimensional groupings should be correct. Expanding the impactor energy and examining the experimental impacts in deep and shallow water the coefficients and exponents can be fitted to obtain

$$\frac{V_i}{V_c} = 1.0 \left(\left(\frac{\rho_c}{\rho_i} \right) \left(\frac{gD}{v^2} \right) \right)^{0.7} + 0.003 \left(\frac{P_0}{\rho_i v^2} \right)^{0.55} + 0.6 \left(\left(\frac{\rho_c}{\rho_i} \right) \left(\frac{gD}{v^2} \right) \left(\frac{D}{H} \right)^2 \right)^{0.5} + 0.01 \left(\left(\frac{P_0}{\rho_i v^2} \right) \left(\frac{D}{H} \right)^{3/2} \right)^{0.35}$$



CONCLUSIONS

- The crater scaling law of Holsapple (1993) has been expanded to include effects of shallow water and background air pressure.
- There is still a discrepancy between the laboratory experiments/simulations and the asteroid impact simulations, which should lie directly on the gravity dominated line.
- Not much experimental data exists in regions greatly dominated by only one of the terms, particularly in the gravity dominated regime where asteroids impacts will be. For example the vapor pressure of water is 2400 Pa which means it is difficult to do an experiment under less than 1/100th of an atmosphere since the water starts to boil. An experiment with ethylene glycol (antifreeze) may be a good choice since it has a vapor pressure of only 7 Pa at room temperature.
- Provided the physics involved in the impact does not change a non-dimensionalized scaling law should be predictive for impacts even well outside the range in which the law is derived. However if the physics involved changes it may no longer be reliable.
- The fit, and confidence in extrapolating laboratory experiments to asteroid impacts, can be improved by considering other energy sinks such as impactor strength which may affect some of the experiments at slower speeds, surface tension which affects very small droplet experiments, and perhaps most importantly vaporization which will affect faster impacts. Only impacts above ~3 km/s will typically vaporize the impactor, and may also vaporize a significant amount of water.